Griffiths

9.5. (a) $f^{\alpha\beta\gamma}$ antisymmetric means for it to be non zero we need $\alpha \neq \beta \neq \gamma$. This gives $8 \times 7 \times 6$ values.

Now ~~any permutatio~~ permuting $f^{\alpha\beta\gamma}$ introduces a $+1$ or -1 factor, so we need to divide by # of permutations

$$\frac{8 \times 7 \times 6}{1 \times 2 \times 3} = 56.$$

(b) $$[\lambda^1, \lambda^2] = \begin{pmatrix} [\sigma_1, \sigma_2] & 0 \\ 0 \; 0 & 0 \end{pmatrix} = \begin{bmatrix} 2i\sigma_3 & 0 \\ 0 \quad 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2i & & \\ & -2i & \\ & & 0 \end{bmatrix}$$

The form above indicates that $[\lambda^1, \lambda^2]$ is only linearly combination of λ^3, which makes it obvious that $f^{12\gamma} = 0$ for $\gamma \neq 3$.

(c) $$[\lambda^1, \lambda^3] = \begin{bmatrix} [\sigma_1, \sigma_3] & 0 \\ & 0 \\ 0 \quad 0 & 0 \end{bmatrix} = \begin{bmatrix} -2i\sigma_2 & 0 \\ & 0 \\ 0 \quad 0 & 0 \end{bmatrix} = -2i\lambda^2$$

$$\Rightarrow f^{13\gamma} = \begin{cases} 0 & \text{if } \gamma \neq 2 \\ \frac{1}{\cancel{2i}}, & \text{if } \gamma = 2, \end{cases}$$

$$[\lambda^4, \lambda^5] = \begin{pmatrix} 2i & & \\ & 0 & \\ & & -2i \end{pmatrix} = 2i\left[\frac{\lambda^3}{2} + \frac{\sqrt{3}}{2}\lambda^8\right]$$

$$\Rightarrow f^{453} = \frac{1}{2}, \quad f^{458} = \frac{\sqrt{3}}{2}.$$